(6 pages) Reg. No. : ....

Code No. : 5761 **Sub. Code : WMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2024.

Second Semester

Mathematics — Core

REAL ANALYSIS — II

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. The measure of an Interval I, denoted by $m(I)$ = __________.

   (a) Initial value (b) Terminal value

   (c) 0 (d) $l(I)$

2. Continuous functions are __________.

   (a) not measurable

   (b) absolutely measurable

   (c) measurable

   (d) partially measurable

3. Let f be measurable. $\inf\{\alpha / f \le \alpha \ a.e\}$ is called __________.

   (a) essential supremum

   (b) supermum

   (c) essential infimum

   (d) balanced infimum

4. If f is a Riemann integral over $[a, b]$ then $\int_a^b f$ __________ $\int_a^b f$.

   (a) $<$ (b) $=$

   (c) $>$ (d) $\neq$

5. If f is integrable then $\left|\int f dx\right|$ __________ $\int |f| dx$.

   (a) $\le$ (b) $<$

   (c) $\ge$ (d) $>$

6. A non negative infinite valued function taking only finite number of values is called __________ function.

   (a) finite (b) measurable

   (c) constant (d) simple

7. The Inner Product of f, g is $\langle f, g\rangle$ = __________.

   (a) $\int \overline{f(x)} g(x) dx$ (b) $\int g(x) dx$

   (c) $\int f(x) \overline{g(x)} dx$ (d) $\int f(x) dx$

8. $\lim_{n\to\infty} \int_0^{2\pi} f(x) \sin nx dx$ = __________.

   (a) 0 (b) 2

   (c) 1 (d) π

9. A function ϕ_n is orthonormal if $\|\phi_n\|$ = __________.

   (a) 0 (b) -1

   (c) 2 (d) 1

10. A function $f : R^n \to R^m$ is linear if $f(ax+by)$ = __________.

    (a) $af(x)+bf(y)$ (b) $af(x)-bf(y)$

    (c) $bf(x)-af(y)$ (d) $bf(x)+af(y)$

11. A function f is differentiable at c if there exists $T_c : R^n \to R^m$ such that $f(c+v)$ = __________.

    (a) $T_c|v| + \|v\|$ (b) $f(c) + T_c|v| + \|v\| E_c(v)$

    (c) $T_c|v| - \|v\| E_c(v)$ (d) $\|v\| E_c(v)$

12. If f is a linear function then $f(c+v)$= __________.

    (a) $f(c)-f(v)$ (b) $f(v)-f(c)$

    (c) $f(v)$ (d) $f(c)+f(v)$

13. The set [0, 1] is __________.

    (a) countable (b) non bounded

    (c) bounded (d) not countable

14. A function f has local extremum at c if,

    (a) $f'(c)>0$ (b) $f'(c)=0$

    (c) $f'(c)<0$ (d) $f'(c)\neq 0$

15. A function $f : s \to T$ from (S, d_S) to (T, d_T) is an open mapping if for A in S, $f(A)$ is __________ in T.

    (a) open (b) closed

    (c) imbedded (d) bounded

PART B — (5 × 4 = 20 marks)

Answer ALL questions by choosing either (a) or (b).

16. (a) If $m^*(A)$ is finite show that $m^*(\phi)=0$.

    Or

    (b) Show that a countable set has outer measure zero.

17. (a) Prove Lebesgue dominated convergence theorem.

Or

(b) State and prove Simple Approximation Lemma.

18. (a) Prove chain rule among functions.

Or

(b) State Riemann Localization Theorem.

19. (a) Let $f : S \to R^n$. Let C be an interior point of S. If f is differentiable at C then show that f is continuous at C.

Or

(b) Derive Parseval's Formula.

20. (a) Prove Bounded Convergence Theorem.

Or

(b) Prove Egoroff's theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).

21. (a) Prove that not every measurable set is a Borel Set.

Or

(b) Prove the outer measure of intervals is its length.

22. (a) Prove monotone convergence theorem on measurable functions.

Or

(b) State and prove Fatou's Lemma.

23. (a) Prove Riemann-Lebesgue Lemma.

Or

(b) State and prove Riesz-Fischer Theorem.

24. (a) Prove Taylor's Formula.

Or

(b) Prove mean value theorem for differentiable functions.

25. (a) State and prove Implicit Function Theorem.

Or

(b) State and prove Vitali's Covering Lemma.

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