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Reg. No. : ....

Code No. : 10729 E **Sub. Code : EEMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective — VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The value of $\vec{i} \cdot \vec{j}$ is

(a) 0 (b) 1

(c) $\vec{k}$ (d) k

2. A vector $\vec{f}$ is called a harmonic vector if

(a) $\nabla\vec{f} = 0$ (b) $\nabla^2\vec{f} = 0$

(c) $\nabla \cdot \vec{f}$ (d) $\nabla \times \vec{f}$

3. $\iiint_D dxdydz$ represents

(a) volume

(b) area

(c) circumference

(d) region

4. The value of $\int_0^1\int_0^2 xy^2 dydx$

(a) $\frac{2}{3}$ (b) 3

(c) 4 (d) $\frac{4}{3}$

5. If C is the straight line joining (0, 0, 0) and (1, 1, 1) then $\int_c \vec{r} \cdot d\vec{r}$ is

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

6. The value of $\int_0^a\int_0^a x dxdy$ is

(a) $\frac{1}{2}$ (b) $\frac{a^3}{2}$

(c) $\frac{a^3}{3}$ (d) $\frac{a^2}{2}$

7. For a closed surface S, $\iint_s \vec{r} \cdot \hat{n}\, ds =$

(a) v (b) $2v$

(c) $3v$ (d) 0

8. Stoke's theorem connects

(a) line and double integral

(b) line and surface integral

(c) double and surface integral

(d) surface and volume integral

9. An example of an even function is

(a) x (b) x^2

(c) x^3 (d) $\sin x$

10. For any integer n, the value of $\cos n\pi$ is

(a) 1 (b) 0

(c) -1 (d) $(-1)^n$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find the unit normal to the surface $x^3 - xyz + z^3 = 1$ at (1, 1, 1).

Or

(b) Show that $div\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$.

12. (a) Evaluate $\int_0^1\int_x^1 (x^2 + y^2)\, dydx$.

Or

(b) Evaluate $\int_0^a\int_0^x\int_0^y xyz\, dzdydx$.

13. (a) Evaluate $\int_c \vec{f} \cdot d\vec{r}$ where $\vec{f} = (x^2 + y^2)\vec{i} + (x^2 - y^2)\vec{j}$ and C is the curve $y = x^2$ joining (0, 0) and (1, 1).

Or

(b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$

14. (a) Show that $\iint_S \vec{f} \cdot \vec{n}\,ds = \iiint_V a^2\,dv$ where $\vec{f} = \phi\vec{a}$ and $\vec{a} = \nabla\phi$ and $\nabla^2\phi = 0$

Or

(b) By using Stoke's theorem prove that $\int_C \vec{r} \cdot d\vec{r} = 0$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

15. (a) Define the following with examples.

(i) odd function

(ii) even function

Or

(b) Find the Fourier constant a_0 for the function $f(x) = e^{-x}$ in the interval $(-1, 1)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) If $\nabla\phi = (y + \sin z)\vec{i} + x\vec{j} + x\cos z\vec{k}$, find ϕ.

Or

(b) If $\vec{f}$ is solenoidal prove that curl curl curl curl and $\vec{f} = \nabla^4\vec{f}$.

17. (a) Evaluate $\int_0^\infty\int_0^\infty \left(\frac{\sigma^{y}}{y}\right) dy\,dx$, $\int_0^\infty\int_0^\infty \left(\frac{e^{-y}}{y}\right) dy\,dx$

Or

(b) Evaluate by $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.

18. (a) Evaluate $\iint_S \vec{f} \cdot \vec{n}\,ds$ where

$\vec{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

Or

(b) Find the work done by the force $\vec{F} = 3xy\vec{i} - 5z\vec{j} + 10x\vec{k}$ along the curve C. $x = t^2 + 1$; $y = 2t^2$; $z = t^3$ from $t = 1$ to $t = 2$.

19. (a) Verify Gauss divergence theorem for $\vec{f} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$; $z = 0$ and $z = h$.

Or

(b) Using Green's theorem $\int_C (xy - x^2)\,dx + x^2y\,dy$ along the closed curve C formed by $y = 0$, $x = 1$, $y = x$.

20. (a) Determine the fourier expansion of the function $f(x) = x$ when $-\pi \le x \le \pi$.

Or

(b) Obtain a cosine series for $f(x) = e^x$ in $0 < x < \pi$.

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